Appendix M Structural Analysis of the Macondo #252 Work String

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**SES Document No.: 1101190-ST-RP-0003**

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**Prepared for:**

Transocean Offshore Deepwater Drilling, Inc.

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Structural Analysis of the Macondo #252 Work String

SES DOCUMENT NO.: 1101190-ST-RP-0003

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MAY 2011
EXECUTIVE SUMMARY

Transocean Offshore Deepwater Drilling, Inc., (Transocean) retained Stress Engineering Services, Inc. (SES) to provide technical assistance in their investigation of the blowout that occurred April 20, 2010. Hydraulic analysis, reported elsewhere, indicates that well flow after 21:39 CDT could result in a net upward force in the work string at the level of the Blowout Preventer (BOP). Transocean requested that SES provide structural modeling of the work string and calculations that may help in understanding the behavior of the work string. Structural modeling, assumptions and results of calculations are presented here.

A record of the lengths of individual components in the work string (called a tally) was not available. Therefore, the structural model is somewhat idealized since individual component lengths and the vertical position of the top tool joint, were not available. The component lengths used in the hydraulics analysis were used in the structural model. Pipe joint lengths for each size were assumed equal. The average length used is close to the nominal length of the pipe joint. The structural modeling results are representative and are suitable for understanding the behavior of the work string under various loading conditions.

Load cases were selected based on estimates of effective compression in the pipe near the BOP. Calculations were performed for effective compression up to 150 kips. Two configurations were analyzed in sequence: (1) all BOP elements in the BOP stack are open, and (2) a simulation of closure of the upper annular and the upper variable bore ram (VBR). At the beginning of the simulation, one 5-1/2” tool joint is in the BOP. Vertical displacement of the tool joint, contact loads between the work string and the riser, BOP and casing, and stresses in the work string are of interest.

The work string deforms into a helical configuration in contact with the inside of the riser, the BOP and the casing. Only a portion of the 6-5/8” (upper section of the work string) deforms into a helical configuration; the amount depends on the force applied.

The calculated vertical displacement of the tool joint in the BOP is less than seven feet. Temperature effects, which were not included, would serve to reduce the vertical displacement.
Calculated contact loads between the work string and the inside of the BOP are less than 10 kips. This is the force that would be required to move the pipe away from the wall of the BOP. Calculated stress in the work string is less than yield for all cases.

DNV’s forensic investigation indicated that the tool joint in the BOP was partially in the upper annular when the pipe was sheared. In the idealized structural model, the tool joint in the BOP does not reach the elevation of the upper annular due to the applied hydraulic loads. However, the discrepancy in elevation is within the error of the idealized model.
LIMITATIONS OF THIS REPORT

The scope of this report is limited to the matters expressly covered. This report is prepared for the sole benefit of Transocean Offshore Deepwater Drilling, Inc. (“Transocean”). In preparing this report, Stress Engineering Services, Inc. (SES) has relied on information provided by Transocean. Stress Engineering Services, Inc. (SES) has made no independent investigation as to the accuracy or completeness of such information and has assumed that such information was accurate and complete. Further, Stress Engineering Services, Inc. (SES) is not able to direct or control the operation or maintenance of client’s equipment or processes.

All recommendations, findings and conclusions stated in this report are based upon facts and circumstances, as they existed at the time that this report was prepared. A change in any fact or circumstance upon which this report is based may adversely affect the recommendations, findings, and conclusions expressed in this report.

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1 INTRODUCTION

Transocean Offshore Deepwater Drilling, Inc., (Transocean) retained Stress Engineering Services, Inc. (SES) to provide technical assistance in their investigation of the blowout that occurred April 20, 2010. Hydraulic analysis, reported elsewhere, indicates that well flow after 21:39 CDT could result in a net upward force in the work string at the level of the Blowout Preventer (BOP). Transocean requested that SES provide structural modeling of the work string and calculations that may help in understanding the behavior of the work string. Structural modeling, assumptions and results of calculations are presented here.

A record of the lengths of individual components in the work string (called a tally) was not available. Therefore, the structural model is somewhat idealized since individual component lengths and the vertical position of the top tool joint, were not available. The component lengths used in the hydraulics analysis were used in the structural model. Pipe joint lengths for each size were assumed equal. The average length used is close to the nominal length of the pipe joint. The structural modeling results are representative and are suitable for understanding the behavior of the work string under various loading conditions.

Load cases were selected based on estimates of effective compression in the pipe near the BOP. Calculations were performed for effective compression up to 150 kips. Two configurations were analyzed in sequence: (1) all BOP elements in the BOP stack are open, and (2) a simulation of closure of the upper annular and the upper variable bore ram (VBR). At the beginning of the simulation, one 5-1/2” tool joint is in the BOP. Vertical displacement of the tool joint, contact loads between the work string and the riser, BOP and casing, and stresses in the work string are of interest.

Numerical modeling of the work string is described in Appendix A. Structural calculations are performed with RAMS, SES’ proprietary software [1]. A simplified beam model of the pipe between the annular and the VBR is described in Appendix B, together with results for the selected load cases. The beam model is similar to that described in the DNV report [3]. Calculations for the load cases are described in Appendix C. Modeling an ideal helix is
presented in Appendix D. Comparisons to numerical modeling and equations that may be useful in associated analytical calculations are presented.
2 WORK STRING MODEL

The work string configuration used for structural analysis is the same as that used for hydraulic analysis [2]. A diagram of the well is in Figure 1, taken from [2]. A record of the lengths of individual components in the work string (called a tally) was not available. The component lengths used in the hydraulics analysis were used in the structural model. Pipe joint lengths for each size were assumed equal.

With these assumptions, the center of the 5-1/2” tool joint in the BOP is at 5,021 ft RKB. Two 5-1/2” drill pipe joints span the BOP (joints 20 and 21 below the 6-5/8”). The vertical location of the tool joint is uncertain, due to uncertainty in joint lengths and uncertainty regarding the vertical position of the top of the model. The model assumes a 6-5/8” tool joint at the drill floor. The tool joint would more likely be a few feet above the drill floor so slips can be set on the pipe. The initial configuration of the work string near the BOP is illustrated in Figure 2. In the figure, the horizontal scale is amplified by a factor of 10. Locations of the elements of the BOP are shown.

Details of the RAMS model are in Appendix A.

A model of only the portion of the work string between the upper annular and the upper VBR was also developed. The configuration is similar to that shown in Figure 127 of [3]. The model does not include the effects of the work string above the upper annular or below the upper VBR. Details of the model are in Appendix B.
Figure 1: Diagram of Macondo #252, April 20, 2010

Note: Only production casing and 9-7/8" liner shown; other casing and liner strings omitted for clarity.
Initial configuration

Thin rings at locations of annulars and rams

Tool Joint in BOP at 5,021' 

Upper Annular
Lower Annular
BSR
CSR
VBR (upper)
VBR (lower)
Test Ram

Note: Lateral (X,Y) scale is 10x vertical scale (Z)

Figure 2: Initial configuration of pipe near BOP
3 LOADING

Prior to closure of the variable bore ram (VBR), flow up the annulus between the work string and the casing produced an upward force on the work string that may have exceeded the weight of the work string below the VBR (see 4.7.6 of [2]). The flow rate, and consequently the upward force, was increasing rapidly. The net upward force would cause the tool joint in the BOP to move up.

When the annular closed, it did not stop the flow and flow rate continued to increase until the VBR was closed [2]. Flow stopped when the VBR closed. The pressure below the VBR increased as the well approached shut in conditions. At the same time, the pressure above the VBR was dropping due to expansion of the hydrocarbons in the riser.

Two sets of load cases were selected: (1) drag loading prior to closure of the VBR, and (2) loading due to the pressure differential across the VBR after closure of the VBR. The load cases are described by the level of compression in the work string at the BOP. The selected load cases are listed in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Compression at BOP, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag</td>
<td>30  60  90  120  150</td>
</tr>
<tr>
<td>VBR</td>
<td>NA  60  90  120  150</td>
</tr>
</tbody>
</table>

Details of the load cases, including associated calculations, are in Appendix C. The range of load cases is intended to bound loads that may have occurred.

Drag loads were applied to the RAMS model, starting with 30 kips and increased to the maximum of 150 kips. To simulate closing the annular, the center of the tool joint in the BOP was moved to the centerline of the BOP. To simulate closing the VBR, a point below the tool joint was moved to the centerline of the BOP. The distance between the two points was selected to represent the distance between the upper annular and the upper VBR (27.3 ft). The load was then decreased from the maximum of 150 kips to the minimum of 60 kips.
The VBR loads were also applied to the reduced model in Appendix B.

4 RESULTS FOR DRAG LOAD CASES

The effective tension distributions for the five drag load cases are in Figure 3 (and Appendix C).

The maximum compression (negative tension) occurs at the first tool joint below the BOP. The pipe that is in compression will contact the wall (of the production casing, the BOP, or the riser) and form a helix. All of the 5-1/2” drill pipe is in compression for all cases. Varying amounts of the 6-5/8” drill pipe are in compression. The deformed shape of 36 joints of the 5-1/2” is illustrated in Figure 4. A closer look near the BOP is in Figure 5. The pitch of the helix is close to the height of the BOP (53 ft).
Deformed shape – 36 joints of 5-1/2”

Drag load case 120 kips at BOP

Figure 4: Deformed Shape of 5-1/2” Drill Pipe

Deformed shape near BOP

Drag load case 120 kips at BOP

Figure 5: Deformed Shape of 5-1/2” Drill Pipe near the BOP
Drag loads on the work string cause the pipe to displace upward. Calculated upward displacements for a range of loads are in Figure 6. Tool joint contact forces are in Figure 7. If the annular closed on a tool joint, then the tool joint was probably a few feet below the annular prior to 21:39. The pipe was in contact with the inside of the BOP, riser and casing. Closing the annular would result in a horizontal force on the annular. If the tool joint restricted further upward motion, the pipe above the annular would retain the helical shape just prior to closure. The pipe below the annular would continue to have increasing compression and be in contact with the wall.

Figure 6: Upward Displacement for a Range of Drag Loads
Figure 7: Tool Joint Contact Loads for a Range of Drag Loads
5 RESULTS FOR VBR LOAD CASES

The effective tension distributions for the four VBR load cases are in Figure 8 (and in Appendix C). The pipe below the VBR hangs under its own weight. The jump due to pressure drop across the VBR is apparent. The maximum compression occurs at the VBR.

Effective tension for VBR load cases

![Diagram of effective tension for VBR load cases]

Assumes 2 ppg above VBR and 5 ppg below VBR

Figure 8: Effective Tension for Four VBR Load Cases

The pipe above the annular has a helical shape and the pipe below the VBR is straight as illustrated in Figure 9. The configuration between the annular and VBR (the two constraints) is illustrated in Figure 10. In the model, the elevation of the tool joint did not reach the upper annular, indicating that the assumed initial position of the tool joint is off by a few feet. The pipe between the annular and the VBR is essentially planar and is in contact with the BOP. The idealized case in Appendix D also indicates that the pipe would be planar between the supports (i.e. the upper annular and the upper VBR) and would be in contact between the supports.
Shape with annular and VBR closed

Load case 120 kips compression at VBR

Figure 9: Shape with Annular and VBR Closed

Configuration near BOP

Shape essentially planar between constraints

Constraints (lateral only)

Figure 10: Shape between Annular and VBR
The wall contact force on the pipe between the annular and the VBR is shown in Figure 11. The load calculated using the reduced model in Appendix B is also in Figure 11 for comparison. The contact load from the full work string model is lower than the contact load from the reduced model in Appendix B (labeled “Beam” in the figure). The load is lower due to the influence of the pipe below the VBR, because the casing tends to centralize the drill pipe.

![Figure 11: Contact Load between Annular and VBR](image-url)
6 CALCULATED STRESS

The maximum calculated axial plus bending stress for the load cases is in Table 2.

Table 2: Maximum Calculated Axial Plus Bending Stress for Load Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>30 kips</th>
<th>90 kips</th>
<th>120 kips</th>
<th>150 kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag</td>
<td>21</td>
<td>62</td>
<td>84</td>
<td>105</td>
</tr>
<tr>
<td>VBR</td>
<td>NA</td>
<td>74</td>
<td>88</td>
<td>108</td>
</tr>
</tbody>
</table>

The maximum stress is in the 5-1/2” drill pipe in the BOP. The specified minimum yield stress is 135 ksi. A difference in internal pressure and external pressure would increase the von Mises stress. To increase the von Mises stress to yield, a difference in internal pressure and external pressure of 10,000 psi, which is greater than difference in internal pressure and external pressure that may have occurred, would be required to yield the pipe. The calculated stress for all cases is below yield.
REFERENCES


Appendix A: Work String Model
The work string is comprised of three segments as listed in Table A.1.

**Table A.1: Work String Length**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length, ft</th>
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<tr>
<td>6-5/8&quot;</td>
<td>4,103</td>
</tr>
<tr>
<td>5-1/2&quot;</td>
<td>3,443</td>
</tr>
<tr>
<td>3-1/2&quot;</td>
<td>821</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>8,367</strong></td>
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The 3-1/2” tubing was not included in the detailed structural model, but was represented by its weight acting at the bottom of the 5-1/2” drill pipe. Drill pipe joints were modeled as equal length segments. The number of joints in the model is in Table A.2.

**Table A.2: Pipe joints in the model**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length</th>
<th># joints</th>
</tr>
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<tr>
<td>6-5/8&quot;</td>
<td>4,103</td>
<td>94</td>
</tr>
<tr>
<td>5-1/2&quot;</td>
<td>3,443</td>
<td>75</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>7,546</strong></td>
<td><strong>169</strong></td>
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Structural properties of the drill pipe (DP) and of the tool joint (TJ) for each size are listed in Table A.3.

**Table A.3: Structural Properties of Pipe and Tool Joint**

<table>
<thead>
<tr>
<th>Component</th>
<th>6-5/8&quot; DP</th>
<th>6-5/8&quot; TJ</th>
<th>5-1/2&quot; DP</th>
<th>5-1/2&quot; TJ</th>
</tr>
</thead>
<tbody>
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<td>Pipe OD, in.</td>
<td>6.625</td>
<td>8.250</td>
<td>5.500</td>
<td>7.000</td>
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<tr>
<td>Pipe wall thickness, t, in.</td>
<td>0.500</td>
<td>1.750</td>
<td>0.361</td>
<td>1.500</td>
</tr>
<tr>
<td>Pipe ID, in.</td>
<td>5.625</td>
<td>4.750</td>
<td>4.778</td>
<td>4.000</td>
</tr>
<tr>
<td>External area, Ao, in²</td>
<td>34.472</td>
<td>53.456</td>
<td>23.758</td>
<td>38.485</td>
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<td>Internal area, Ai, in²</td>
<td>24.850</td>
<td>17.721</td>
<td>17.930</td>
<td>12.566</td>
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<tr>
<td>Steel area, A, in²</td>
<td>9.621</td>
<td>35.736</td>
<td>5.828</td>
<td>25.918</td>
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<td>Axial stiffness, AE, lb</td>
<td>2.8863E+08</td>
<td>1.0721E+09</td>
<td>1.7485E+08</td>
<td>7.7754E+08</td>
</tr>
<tr>
<td>Bending stiffness, EI, lb-ft²</td>
<td>9.4622E+06</td>
<td>4.2168E+07</td>
<td>4.0281E+06</td>
<td>2.1936E+07</td>
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For gravity loading, the weight of the pipe, the volume inside the pipe and the volume displaced by the pipe are needed. The weight per unit length of a joint of pipe is assumed constant along the length; the weights used are averaged over a joint. The weights and volumes are in Table A.4.

**Table A.4: Pipe Weight and Volume**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length</th>
<th>Weight, lb/ft</th>
<th>Internal</th>
<th>External</th>
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<tr>
<td>6-5/8&quot;</td>
<td>4,103</td>
<td>37.71</td>
<td>1.273</td>
<td>1.85</td>
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<tr>
<td>5-1/2&quot;</td>
<td>3,443</td>
<td>23.9</td>
<td>0.916</td>
<td>1.282</td>
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<td>3-1/2&quot;</td>
<td>821</td>
<td>9.3</td>
<td>0.3652</td>
<td>0.5072</td>
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</table>

To model a tool joint, one element for each the pin and box was used. The element length is 5% of the joint length and the element stiffness varies linearly over the element from the tool joint stiffness to the pipe stiffness. The remaining 90% of the length has uniform properties. Ten equal length elements were used for 34 joints of the 5-1/2”, 18 elements were used for the two joints that span the BOP, and eight elements were used for the lower 39 joints. This gives a total of 738 elements to model the 5-1/2”.

A similar model was developed for the 6-5/8”. The lower 65 joints were modeled using 5% of the length for pin and box, with four equal length elements for the uniform segment. Fewer elements were used for the upper portion of the 6-5/8”. The total number of elements in the 6-5/8” model is 450.

Pipe contact with the inside of the riser, BOP or casing was modeled using distributed quadratic springs (see Appendix D). The hole diameter in the model is in Table A.5. The riser inside diameter is 19.5”, the BOP inside diameter is 18.75”, and the casing inside diameter is 8.625”. Changes in diameter were assumed to vary linearly over ten feet.
Model parameters were determined as described in Appendix D. The spring stiffness used is the same and the initial contact radius is reduced by 0.015 ft as in Appendix D.

Tool joints contact the hole first due to their larger diameter. Contact was modeled by a single quadratic spring in the center of the tool joint. The spring is equivalent to a one foot length of the distributed springs.

The 3-1/2” tubing was modeled as a vertical force at the bottom of the 5-1/2”. Assuming the tubing is filled with seawater and seawater is in the annulus, the calculated force (weight of tubing) is 6,638 lb. In the initial condition, the work string and annulus are assumed filled with seawater.

The top of the work string is pinned. The vertical location is 0 ft RKB.
Appendix B: Beam Model
Modeling the pipe between the annular and the VBR as a beam can provide useful information. The effects of pipe above the annular and below the VBR are ignored. The pipe is assumed uniform with properties of the 5-1/2” pipe (tool joints are not modeled). The model is assumed weightless. The model length is 27.3 ft (see Figure 127 of [3]).

The model results presented here assumed the ends of the beam are pinned. One end is axially restrained and a vertical load is applied at the other end. If the axial load is less than the Euler buckling load, then the idealized model is straight. For higher loads, the beam will deflect and contact the inside of the BOP. Contact is modeled using the same distributed quadratic springs as used in the work string model.

The calculated Euler buckling loads for various boundary conditions are listed in Table B.1. A pinned boundary condition provides no rotational restraint. A clamped boundary condition allows no rotational displacement. The estimated critical load in the DNV report [3] is 113,568 lb, which is close to the clamped-pinned case.

<table>
<thead>
<tr>
<th>Case</th>
<th>Euler Buckling Load, lb</th>
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<tr>
<td>pinned-pinned</td>
<td>53,343</td>
</tr>
<tr>
<td>clamped-pinned</td>
<td>108,820</td>
</tr>
<tr>
<td>clamped-clamped</td>
<td>213,363</td>
</tr>
</tbody>
</table>

The total contact force acting on the inside of the BOP is shown in Figure B.1 for a range of loads. For comparison, the loads calculated from the work string model are also shown. The maximum calculated stress is shown in Figure B.2.
Figure B.1: Contact Force on BOP for A Range Of Loads

Figure B.2: Maximum Stress for a Range of Loads
The calculated contact force from the beam model is higher than the contact force from the work string model.

Additional comparisons were made to the model in Appendix D (an ideal helical model). The model in Appendix D is 20 times the length of the beam model. A 27.3 ft portion of the model in Appendix D was constrained by moving the nodes to the centerline. A comparison of the distributed contact load is in Figure B.3 and a comparison of stress is in Figure B.4.

![Figure B.3: Comparison of Contact Loads for F=150 kips](image)
Figure B.4: Comparison of Stress for F=150 kips
Appendix C: Discussion of Loads
Loads on the work string can come from drag loads due to flow up the annulus or from a pressure differential across a closed BOP ram. The high flow rate conditions prior to closure of the variable bore ram could produce frictional pressure losses in the annulus between the drill pipe and the casing. The resulting vertical load on the work string can exceed the weight of the work string below the BOP. The resulting compressive load in the work string can lead to helical buckling and upward displacement of the work string. Closing the variable bore ram could stop the flow, resulting in a pressure increase below the ram as the well reaches a shut in condition. Calculations associated with load estimates are presented here.

Consider a segment of pipe suspended in casing as illustrated in Figure C.1. The pipe is closed at the top and is open at the bottom. The initial condition is static. The force, $F$, required to support the pipe is equal to the weight of the pipe plus the weight of the fluid in the pipe less the weight of the fluid displaced by the pipe. The pressures inside and outside the pipe at the top may be different if the fluid in the pipe and the fluid in the annulus have different densities. The force, $F$, is called the effective tension. The weight of the pipe plus the weight of the fluid in the pipe less the weight of the fluid displaced by the pipe is called the effective weight of the pipe.

![Figure C.1: Pipe Suspended in Casing](image)
The first case we consider is such that there is flow up the annulus and the pressure outside the pipe at the top is constant. Flow up the annulus results in a frictional pressure drop in the annulus. The change in the force to support the pipe consists of two components: (1) the pressure end load, and (2) friction on the pipe due to the flow. The changes in force and pressure are:

\[
\Delta P_{friction} \quad \text{is the frictional pressure drop}
\]
\[
\Delta P_o \quad \text{is the change in back pressure}
\]
\[
\Delta P_i = \Delta P_{friction} + \Delta P_o \quad \text{is the change in internal pressure}
\]
\[
\Delta F = -\Delta P_{friction} \left( \frac{\pi d^2}{4} + \frac{\pi (D^2 - d^2)}{4} \frac{d}{D+d} \right) \quad \text{is the change in force}
\]
\[
d \quad \text{is the diameter of the pipe}
\]
\[
D \quad \text{is the inside diameter of the casing}
\]

The first term in the expression for change in force is the pressure end load and the second term is friction on the pipe. In the second term, the assumption is made that the average shear stress on the surface area of the annulus is applied to the surface area of the pipe. If the density of the fluid in the annulus changes, there is an additional component of change in force due to change in effective weight of the pipe.

![Figure C.2: Pipe Suspended in Casing with Annular Seal](image-url)
The second case assumes a seal in the annulus, so there is no flow in the annulus as shown in Figure C.2. Prior to closure of the seal, the pressure below the seal and the pressure above the seal are equal. The changes in force and pressure are:

\[
\begin{align*}
P_o & \quad \text{is the pressure above the seal} \\
P_{VBR} & \quad \text{is the pressure below the seal} \\
\Delta P_{VBR} = P_{VBR} - P_o & \quad \text{is the pressure drop across the seal} \\
\Delta F = -\Delta P_{VBR} \left( \frac{\pi d^2}{4} \right) & \quad \text{is the change in force} \\
d & \quad \text{is the pipe diameter}
\end{align*}
\]

The load is equivalent to a vertical force applied at the location of the seal. The seal is assumed frictionless and does not provide vertical restraint.

When the change in force exceeds the effective weight, the pipe is in effective compression. We can use these formulae to estimate the pressure drop to produce a given net effective compression.

For the drag loading case, we assume that the frictional pressure drop occurs in the drill pipe/casing annulus below 5,067 ft MD (the first tool joint below the BOP). The effective weight of the work string below 5,067 ft is 73 kips, which assumes water inside the work string and 4 ppg hydrocarbon in the annulus. The 5-1/2” drill pipe diameter was modified to an equivalent diameter to account for the tool joints in the pressure drop calculations.

For the VBR loading case, the effective weight of the work string below the VBR is 70 kips, which assumes water in the work string and 5 ppg hydrocarbon in the annulus. We use the nominal pipe diameter for the area calculation since the VBR is assumed to close on the pipe (not on a tool joint).

The dimensions used for calculation of pressure drop are in Table C.1. The resulting calculated pressure drop for a range in net compression at the BOP is in Figure C.3.
Table C.1: Pressure Drop Calculation Parameters

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>5.500</td>
<td>in</td>
</tr>
<tr>
<td>Adjusted d</td>
<td>5.606</td>
<td>in</td>
</tr>
<tr>
<td>D</td>
<td>8.625</td>
<td>in</td>
</tr>
<tr>
<td>Drag</td>
<td>38.0</td>
<td>lb/psi</td>
</tr>
<tr>
<td>VBR</td>
<td>23.76</td>
<td>lb/psi</td>
</tr>
</tbody>
</table>

When the VBR is closed, the pressure below the VBR increases and the pressure above the VBR will decrease as the hydrocarbons expand in the riser. In the shut in condition, the pressure below the VBR is 8,000 – 8,500 psi. With an assumed hydrocarbon density of 2 ppg above the VBR, the pressure above the VBR is 500 psi. Thus, the pressure drop across the VBR is about 8,000 psi, which corresponds to a net compression of about 120 kips. For structural analysis, loads from 60 kips to 150 kips were used. A vertical load was applied to the model at 5,049 ft MD.
For the drag loading case, we apply a uniformly distributed force from 5,067 ft to 7,546 ft (bottom of the 5-1/2” drill pipe). For structural analysis, loads from 30 kips to 150 kips were used.

The effective tension distributions for the selected load cases are in Figures C.4 and C.5.
Figure C.5: Effective Tension Distribution for VBR Load Cases
Appendix D: Modeling a Helix
The equations for the centerline coordinates of a helix are

\[
\begin{align*}
X &= r \cos \left( \frac{2 \pi s}{p} \right) \\
Y &= r \sin \left( \frac{2 \pi s}{p} \right) \\
Z &= s\sqrt{1 - \left( \frac{2 \pi s}{p} \right)^2}
\end{align*}
\]

where
\( r \) is the radius of the helix
\( p \) is the pitch of the helix
\( s \) the the distance along the centerline

For a uniform, weightless pipe inside a cylindrical hole, the equilibrium configuration can be determined analytically. The equations are

\[
\begin{align*}
r &= \frac{D - d}{2} \quad \text{is the radius of the helix} \\
p &= \sqrt[3]{\frac{8 \pi^2 EI}{F}} \quad \text{is the pitch of the helix} \\
M &= \frac{Fr}{2} \quad \text{is the bending moment} \\
N &= \frac{F^2 r}{4EI} \quad \text{is the wall contact force per unit length} \\
D &= \text{the diameter of the hole} \\
d &= \text{the diameter of the pipe} \\
EI &= \text{the bending stiffness of the pipe} \\
F &= \text{the compressive force along the axis of the hole}
\end{align*}
\]

We compare the analytical solution to results from an equivalent numerical model analyzed in RAMS. The analytical formulation may also be used as a check against the numerical results presented in the main body of the report.

For numerical modeling, we use 5-1/2” drill pipe inside an 18-3/4” hole. We assume uniform properties and assume the pipe is weightless to fit the analytical model. One end is pinned and the other is fixed laterally (X=Y=0) and a compressive force in the Z direction is applied. The
model length is $20 \times 27.3 = 546$ ft to provide sufficient length for developing the helical form away from the ends.

Radial restraint is provided by distributed quadratic springs. The radial force is given by

$$N = \begin{cases} K(r - r_0)^2 & r > r_0 \\ 0 & r < r_0 \end{cases}$$

$r$ is the radial displacement of the pipe

$r_0$ is the initial contact with the spring

Model parameters are summarized in Table D.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>18.75 in.</td>
</tr>
<tr>
<td>$d$</td>
<td>5.5 in.</td>
</tr>
<tr>
<td>$EI$</td>
<td>$4.028E+06$ lb-ft$^2$</td>
</tr>
<tr>
<td>Length</td>
<td>546 ft</td>
</tr>
<tr>
<td>$K$</td>
<td>$1.00E+07$ lb/ft/ft$^2$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>0.537 ft</td>
</tr>
<tr>
<td>Ideal $r$</td>
<td>0.552 ft</td>
</tr>
</tbody>
</table>

The initial contact with the radial spring is 0.015 ft less than the ideal radius.

The model was analyzed for loads over the range of 30 kips to 150 kips.

The $X$ displacement for a range of loads is in Figure D.1. There is a transition from the pinned end to first contact with the wall of the hole. The middle portion of the model is helical. The pitch of the helix decreases with increasing load. The calculated stress is in Figure D.2. The transition at each end is apparent. The middle, helical portion has constant stress.

The pitch, radial displacement, contact load and stress in the middle 50% of the model length are compared to the analytical solution in Figures D.3 – D.6. The error in pitch length, which does not depend on the radial displacement, is much less than 1%. The pitch ranges from 103 ft to 46 ft. The model radial displacement varies slightly with load due to the radial spring constraint.
The slight discrepancy in radial displacement results in a corresponding discrepancy in contact load and stress since the contact load and the bending moment are proportional to the radial displacement. Agreement between the numerical model and the analytical solution is very good.
Figure D.1: X Displacement for a Range of Loads

Figure D.2: Axial Plus Bending Stress for a Range of Loads
Figure D.3: Comparison of Numerical to Analytical Pitch Length

Figure D.4: Comparison of Numerical to Analytical Radial Displacement
Figure D.5: Comparison of Numerical to Analytical Contact Force

Figure D.6: Comparison of Numerical to Analytical Axial plus Bending Stress
Another Test Case

To examine a somewhat idealized case of closure of the annular and of the variable bore ram, two nodes, separated by 27.3 ft, in the middle of the model, were constrained to the centerline (X=Y=0). Only the case for F=150 kips is presented.

A plan view (X-Y) of the portion of the model between the constrained nodes is in Figure D.7. Prior to setting the constraints, the model is in a helix (as evident by the circular shape). After setting the constraints, the model is planar between the constrained nodes.

Contact loads before and after setting the constraints are in Figure D.8. The contact loads for the helix are constant. After setting the constraints, the model deflects and contacts the wall of the hole. The maximum contact load is much higher than the contact load for the helical configuration.

Axial plus bending stress before and after setting the constraints are in Figure D.9. Stress is constant for the helix. After setting the constraints, the maximum stress is lower.

![Figure D.7: Plan View (X-Y) Before and After Setting Constraints](image-url)
Displacements along Axis of Helix

When a compressive force $F$ is applied, the distance between the ends is reduced. The length of the pipe, measured along the axis of the pipe, is reduced by compressive strain. The distance between the ends, measured along the axis of the hole, is reduced due to the helical shape. The change in distance between the ends is

$$\Delta = \Delta_{\text{axial}} + \Delta_{\text{helix}}$$

is the total change in distance between the ends

$$\Delta_{\text{axial}} = \frac{FL}{AE} \alpha \approx \frac{FL}{AE}$$

is the change due to axial compression in the pipe

$$\Delta_{\text{helix}} = (1 - \alpha)L \approx \frac{Fr^2L}{4EI}$$

is the change due to the helical shape

$$\Delta \approx \Delta_{\text{axial}} \left(1 + \frac{r^2}{4} \frac{AE}{EI}\right)$$

is the total change in distance

$L$ is the undeformed length of pipe

$AE$ is the axial stiffness of the pipe

$EI$ is the bending stiffness of the pipe

$r$ is the radius of the helix

$F$ is the compressive applied load

$$\alpha = \frac{dZ}{ds} = \sqrt{1 - \left(\frac{2\pi r}{p}\right)^2}$$

is approximately 1 for cases considered here

From the fourth equation, the displacement is the axial compression times a constant. For the parameters in this example (5-1/2” pipe in an 18-3/4” hole), the constant is 4.3. Most of the displacement along the axis of the hole is due to the helical shape.

The formulae presented are for a weightless pipe. For a pipe in a vertical hole, the displacement due to the helical shape changes due to variation in the compressive load

$$\Delta_{\text{helix}} = \begin{cases} 
\frac{F^2 - r^2}{8EIw} & F < wL \\
\left(F^2 - (F - wL)^2\right) \frac{r^2}{8EIw} & F > wL \\
w & \text{is the weight of the pipe per unit length}
\end{cases}$$